A slave-fermion gauge-theory approach of the t-J model: Doping-induced complex magnetic structure and Z_2 spin-gapped anomalous metal in an antiferromagnetic doped Mott insulator

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We reinvestigate a doped antiferromagnetic Mott insulator based on the slave-fermion approach of the t-J model, where antiferromagnetic spin fluctuations and doped holes are described by bosonic spinons and fermionic holons, respectively. Earlier studies have shown that an effective field theory for the doped antiferromagnetic Mott insulator is given by a non-relativistic fermion (holon) U(1) gauge theory for charge dynamics and a relativistic boson (spinon) U(1) gauge theory for spin dynamics, thus allowing an anomalous metallic phase where bosonic spinons are gapped away from an antiferromagnetic state, analogous to the U(1) spin liquid phase in the slave-boson approach of the t-J model. We argue that the emergent U(1) gauge structure in this approach is based on a simplified picture for antiferromagnetic correlations. Considering that dynamics of doped holes frustrates a collinear antiferromagnetic spin configuration, we show that doped holes enhance ferromagnetic spin correlations and result in a complex magnetic structure. The presence of such a complex spin texture reduces the U(1) gauge structure down to Z_2, thus a different effective field theory results for the spin-gapped non-Fermi liquid metal at low temperatures because there are no gapless U(1) gauge fluctuations.

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I. INTRODUCTION

Study on strongly correlated electrons opened a new window so called gauge theory in modern condensed matter physics. When electrons are weakly correlated, that is, kinetic energy is more dominant than potential energy, a local order parameter approach is available based on decoupling of interaction channels. This is the heart of "classical" condensed matter physics composed of Fermi liquid theory and Landau-Ginzburg-Wilson framework. On the other hand, when interactions are strong enough compared with kinetic energy, an infinite interaction limit can be a good starting point. The presence of such a large energy scale gives rise to a constraint in dynamics of electrons. In addition, interesting physics now arises from the kinetic-energy contribution in the restricted Hilbert space, and "non-local" order parameters or more carefully, link variables instead of on-site ones in lattice models appear as important low energy collective degrees of freedom. These link variables can be formulated as gauge fields, and gauge theory arises naturally for dynamics of strongly correlated electrons.

In the present study we revisit a doped Mott insulator problem based on one possible gauge theory approach of the slave-fermion representation. The slave-fermion representation relies on the fact that a spin degree of freedom of an electron is represented by a bosonic matter, thus it is advantageous in describing magnetic ordering of localized moments via condensation of bosonic spinons while it has difficulty in connection with Fermi liquid. On the other hand, the slave-boson approach resorts to the fact that a charge degree of freedom of an electron is represented by a bosonic matter, thus the slave-boson gauge theory has its clear connection with the Fermi liquid theory via condensation of bosonic holons while it does not have a "natural" description for ordering of localized magnetic moments, compared to the slave-fermion approach.[1] An immediate question is about statistics of spinons. At present, there is no clear connection between the fermionic and bosonic descriptions for spin degrees of freedom.[2]

The problem to determine which of the representations is available seems to be associated with the nature of an undoped parent Mott insulating phase. Although there is no obvious way to classify undoped Mott insulating phases, it is natural to consider symmetric and symmetry-broken Mott insulators.[3] Symmetric Mott insulators are usually called spin liquids, where both fermionic[4] and bosonic[5] spin descriptions are available. Symmetry-broken Mott insulators can be discriminated further, depending on their symmetry breaking patterns. An example of translational symmetry-broken Mott insulators is a Mott insulator with a valence bond solid order, and that of rotational symmetry-broken ones is an antiferromagnetic Mott insulator. Bond-operator formalism[6] may be useful for the description of the valence-bond-solid Mott insulator while the antiferromagnetic Mott insulator will be described well by the Schwinger-boson representation[7].

In this paper we focus on doping to an antiferromagnetic Mott insulator based on the slave-fermion approach. In early days single-hole dynamics in the antiferromagnetically correlated spin background was one of the main interests.[8] Hole doping frustrates an antiferromagnetic spin configuration. Interactions with such antiferromagnetic fluctuations give rise to a self-energy correction of a

single-hole, resulting in the fact that it resides in four diagonal momentum points of $(\pm\pi/2, \pm\pi/2)$. Mathematically speaking, the hopping term in the slave-fermion representation of the t-J model, usually called the Shraiman-Siggia term in the continuum approximation, plays a crucial role for the single-hole dynamics. In the present paper the role of this term is critically reinvestigated, and we find that it plays an important role for dynamics of both holes and spin fluctuations.

Recently, this problem was revisited by several authors, claiming that the slave-fermion approach allows an anomalous metallic phase with short-range antiferromagnetic correlations, where bosonic spinons are gapped away from the antiferromagnetic Mott insulating phase.[9, 10] An effective field theory for the doped antiferromagnetic Mott insulator was argued to be a nonrelativistic fermion (holon) U(1) gauge theory for charge dynamics (of gapless charged fermions around the four diagonal points) and a relativistic boson (spinon) U(1) gauge theory for spin dynamics (the CP^1 gauge theory of the O(3) nonlinear σ model for antiferromagnetic spin fluctuations), allowing an anomalous metal phase when bosonic spinons are gapped.[11] The presence of such an anomalous metallic phase is attributed to deconfinement[12, 13, 14] of compact U(1) gauge theory, where the presence of gapless fermionic matters is expected to suppress monopole (skyrmion[15]) excitations in two space and one time dimensions $[(2+1)D]$ although the pure compact U(1) gauge theory does not allow such a deconfinement phase in $(2+1)D$[16].

From a microscopic point of view, the emergent U(1) gauge structure can be allowed when hopping of spinons and holons between next nearest neighbor sites, i.e., hopping between same sublattices in the square lattice has its nonzero vacuum expectation amplitude while hopping between different sublattices (nearest neighbor sites) vanishes in the presence of short-range antiferromagnetic correlations (spinon singlet-pairing order). In the present study we reexamine the spinon-holon exchange hopping term carefully, and reveal that the emergent gauge structure is Z_2 instead of U(1) owing to the fact that hopping of doped holes frustrates the collinear antiferromagnetic spin configuration and results in a complex magnetic structure. Such a complex spin texture is reflected in nonzero expectation values of nearest-neighbor hopping parameters of spinons and holons. In other words, dynamics of doped holes enhances ferromagnetic spin fluctuations, reducing the U(1) gauge symmetry, arising in the case when only collinear antiferromagnetic correlations are considered, down to Z_2.

For definiteness, we first consider the antiferromagnetic Heisenberg model based on the Schwinger-boson representation. In this study we allow both ferromagnetic (exchange hopping of spinons between different sublattices) and antiferromagnetic (singlet pairing of spinons between different sublattices) correlations. In addition, emergence of a flux is admitted for the ferromagnetic exchange-hopping channel. Based on the saddle-point analysis for magnetic ordering, we find; (1) If no flux is allowed in the exchange-hopping parameter, the vacuum expectation value of the hopping parameter vanishes, thus our mean-field analysis recovers the conventional Schwinger-boson mean-field theory for collinear antiferromagnetic ordering.[7] The resulting effective field theory is a relativistic U(1) gauge theory, i.e., the CP^1 gauge theory of the O(3) nonlinear σ model.[17] On the other hand, (2) if π-flux is allowed in the hopping channel, the hopping parameter between the different sublattices has its nonzero vacuum expectation value because the presence of π-flux enhances ferromagnetic correlations. As a result, the U(1) gauge symmetry is down to Z_2 since both spin-singlet pairing and exchange hopping exist. The presence of both antiferro- and ferro- magnetic contributions causes the magnetic ordering structure to be complicated. We discuss physics of this possible complex magnetic structure.

Next, we study a doped antiferromagnetic Mott insulator based on the U(1) slave-fermion representation of the t-J model. As discussed before, the main point is how to take the hopping-t term into account. Performing the similar mean-field analysis as the above, we find that the hopping parameter does not vanish even in the uniform flux case owing to the fact that mobile doped holes increase ferromagnetic correlations. As a result, a complex magnetic structure such as a spiral type is expected to appear, consistent with doping-induced spiral magnetic ordering of the previous slave-fermion studies.[8] Enhancement of ferromagnetic correlations induced by doped holes causes a serious result that gapless U(1) gauge fluctuations do not exist in the long-wave length and low-energy limits even when such bosonic spinons become gapped to cause the fact that the spiral-type complex magnetic structure disappears away from half filling. As a result, this spin-gapped anomalous metallic state, where only charge fluctuations are gapless exhibiting so called spin-charge separation, is described by a Z_2 gauge theory instead of a U(1) gauge theory. In this respect such a deconfinement phase is certainly possible for $(2+1)D$, considering that even the pure Z_2 gauge theory without matters allows its deconfinement phase in $(2+1)D$.[18] We propose this gauge-symmetry reduction in the low energy limit as one possible mechanism for spin-charge separation in the doped antiferromagnetic Mott insulator.

II. ANTIFERROMAGNETIC MOTT INSULATOR

A. Schwinger-boson mean-field analysis of the antiferromagnetic Heisenberg model

We consider the quantum antiferromagnetic Heisenberg Hamiltonian

$$\frac{H}{J} = \sum_{ij} \vec{S}_i \cdot \vec{S}_j. \tag{1}$$

Generally speaking, one can decompose the spin-exchange interaction $\vec{S}_i \cdot \vec{S}_j$ with Schwinger-bosons as follows,

(i) Néel scheme,

$$\vec{S}_i \cdot \vec{S}_j = -2\Delta_{ij}^\dagger \Delta_{ij} + S^2 \tag{2}$$

(ii) Spiral scheme,

$$\vec{S}_i \cdot \vec{S}_j = -\Delta_{ij}^\dagger \Delta_{ij} + \chi_{ij}^\dagger \chi_{ij} - S/2 \tag{3}$$

where $\Delta_{ij} = (b_{i\uparrow}b_{j\downarrow} - b_{i\downarrow}b_{j\uparrow})/2$ and $\chi_{ij}^\dagger = (b_{i\uparrow}^\dagger b_{j\uparrow} + b_{i\downarrow}^\dagger b_{j\downarrow})/2$ are associated with antiferromagnetic and ferromagnetic correlations, respectively.

In order to clearly show physical meaning of Δ_{ij} and χ_{ij}, it is convenient to consider classical spins. If we choose two nearest neighbor spins

$$b_i = \sqrt{2S}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad b_j = \sqrt{2S}\begin{pmatrix} e^{-i\phi}\cos(\theta/2) \\ \sin(\theta) \end{pmatrix}, \tag{4}$$

where (ϕ, θ) are spherical coordinates of spin j, then

$$\Delta_{ij} = S\sin(\theta/2) \tag{5}$$

$$\chi_{ij} = Se^{i\phi}\cos(\theta/2). \tag{6}$$

For a ferromagnetic state ($\theta = 0$), we have $\Delta = 0$ and $|\chi| = S$ while in the Néel state of $\theta = \pi$, $\Delta = S$ and $\chi = 0$ result. In this respect Δ and χ represent antiferromagnetic and ferromagnetic correlations, respectively. Furthermore, for classical antiferromagnetic spins, when spins are non-collinear (*i.e.* $\theta \neq \pi$) where neighboring spins have their finite overlap, χ becomes nonzero so that χ describes canting and spiral. One may expect that competition between antiferromagnetic Δ and ferromagnetic χ correlations will give rise to non-collinear ordering in the ground state of antiferromagnetically correlated spins.

Inserting Eqs. (5) and (6) into Eqs. (2) and (3), we get

$$(\vec{S}_i \cdot \vec{S}_j)_{N\acute{e}el} = S^2\cos\theta \tag{7}$$

$$(\vec{S}_i \cdot \vec{S}_j)_{Spiral} = S^2\cos\theta - S/2. \tag{8}$$

It is obvious that the Spiral scheme is more favorite than the Néel scheme in the classical limit since it has lower ground state energy. In the following we focus on the Spiral scheme.

Taking into account the constraint of $\sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} = 2S$ at each site via a Lagrange multiplier field λ_i, an effective Hamiltonian in the Schwinger-boson representation of the antiferromagnetic Heisenberg model can be written as follows

$$\begin{aligned} \frac{H}{J} &= -\sum_{ij,\sigma}\left(\Delta_{ij}^*\sigma b_{i\sigma}b_{j\bar{\sigma}} + h.c.\right) + \sum_{ij,\sigma}\left(\chi_{ij}b_{i\sigma}^\dagger b_{j\sigma} + h.c.\right) \\ &+ \sum_{i,\sigma}\lambda_i(b_{i\sigma}^\dagger b_{i\sigma} - 2S) + \sum_{ij}\left(|\Delta_{ij}|^2 - |\chi_{ij}|^2\right). \end{aligned} \tag{9}$$

It is important to notice that the sign of the spinon-hopping term is positive, implying that spinon-hopping contributions are energetically unfavorable, thus its amplitude will vanish in the uniform-flux case, as will be shown below. Physically speaking, ferromagnetic correlations frustrating the antiferromagnetically correlated background are prohibited in the naive mean-field analysis. However, introduction of π-flux weakens such frustration effects, ferromagnetic correlations contributing to physics of antiferromagnetism.

For a mean-field analysis we replace Δ_{ij} and χ_{ij} with their saddle-point values. In the flux ansatz we parameterize Δ_{ij} and χ_{ij} with

$$\Delta_{ij} = \Delta, \quad \chi_{ij} = \chi e^{i\theta_{ij}} \tag{10}$$

where $\theta_{ij} = \pm\theta$ if ij along or against an arrow in Fig. 1. Note that a flux is allowed only in the ferromagnetic hopping channel. Based on this mean-field ansatz with $\lambda_i = \lambda$, we obtain the following effective Hamiltonian in the momentum space,

$$\begin{aligned} \frac{H}{J} &= -\frac{z\Delta}{2}\sum_{k,\sigma}\left(\sigma\gamma_k b_{k\sigma}b_{\bar{k}\bar{\sigma}} + h.c.\right) + \lambda\sum_{k\sigma}b_{k\sigma}^\dagger b_{k\sigma} \\ &+ z\chi{\sum_{k,\sigma}}' \begin{pmatrix} b_{k\sigma}^\dagger & b_{k+Q\sigma}^\dagger \end{pmatrix}\begin{pmatrix} \gamma_k\cos\theta & i\phi_k\sin\theta \\ -i\phi_k\sin\theta & -\gamma_k\cos\theta \end{pmatrix}\begin{pmatrix} b_{k\sigma} \\ b_{k+Q\sigma} \end{pmatrix} \\ &- 2NS\lambda + \frac{z}{2}N\Delta^2 - \frac{z}{2}N\chi^2 \end{aligned} \tag{11}$$

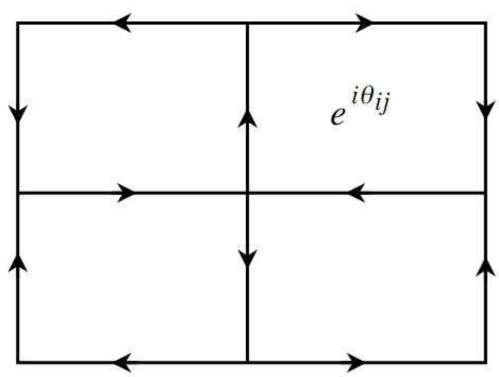

FIG. 1: Flux ansatz

with $\gamma_k = 1/z\sum_{j\in i}(e^{ik_x} + e^{ik_y})$ and $\phi_k = 1/z\sum_{j\in i}(e^{ik_x} - e^{ik_y})$. z is a lattice-coordination number, here $z = 4$ for the square lattice. The $'$ symbol in the momentum summation represents to perform the summation in the folded Brillouin zone.

Performing the Bogoliubov transformation, we can diagonalize the effective Hamiltonian as follows,

$$\frac{H}{J} = \sum_{k,s=\pm}{}^{'} \omega_k^s \left(\alpha_{ks}^\dagger \alpha_{ks} + \beta_{ks}^\dagger \beta_{ks} + 1\right) - 2N\lambda(S+\frac{1}{2}) + \frac{z}{2}N\Delta^2 - \frac{z}{2}N\chi^2, \tag{12}$$

where the quasiparticle spectrum is

$$\omega_k^s = \sqrt{(\lambda + sz\chi\Theta_k)^2 - (z\Delta\gamma_k/2)^2} \tag{13}$$

with $\Theta_k = \sqrt{(\gamma_k\cos\theta)^2 + (\phi_k \sin\theta)^2}$ in the flux ansatz. Then, the resulting mean-field free energy is given by

$$F_{MF} = \frac{2}{\beta}\sum_{ks}{}^{'} \ln\sinh\frac{\beta J\omega_k^s}{2} -2NJ\lambda(S+\frac{1}{2}) + \frac{z}{2}NJ\Delta^2 - \frac{z}{2}NJ\chi^2. \tag{14}$$

Minimizing F_{MF} with respect to Δ, χ, and λ, we obtain self-consistent mean-field equations

$$S + 1/2 = \frac{1}{N}\sum_{ks}{}^{'}\frac{\lambda + s\chi\Theta_k}{\omega_k^s}(n_k + 1/2) \tag{15}$$

$$\Delta = \frac{1}{N}\sum_{ks}{}^{'}\frac{z\Delta\gamma_k^2}{2\omega_k^s}(n_k + 1/2) \tag{16}$$

$$\chi = \frac{1}{N}\sum_{ks}{}^{'}\frac{2zs\Theta_k(\lambda + s\chi\Theta_k)}{\omega_k^s}(n_k + 1/2) \tag{17}$$

with $\Delta \to z\Delta/2$ and $\chi \to z\chi$.

Flux states of our interest are uniform-flux ($\theta = 0$) and π-flux ($\theta = \pi/4$) phases since they do not break time-reversal symmetry. *For the uniform phase*, our numerical analysis shows that the lowest free energy is achieved by $\Delta = 1.1580$ and $\chi = 0$. One can easily check that $\chi = 0$ is a solution for the above self-consistent equations in the uniform-flux case since the right-hand-side of Eq. (17) is odd for momentum k if $\chi = 0$, thus it vanishes after the momentum integration, consistent with the left-hand-side of Eq. (17). In addition, this result is consistent with our physical picture associated with the sign of the ferromagnetic hopping term. Schwinger bosons are condensed to the $k^* = (0, 0)$ state, which corresponds to the collinear antiferromagnetic order. Staggered magnetization is found to be $m_0 = 0.3034$, which is the same as the Auerbach's result[7]. The ground state energy per bond is given by

$$E_0 = -\Delta^2 = -0.3352. \tag{18}$$

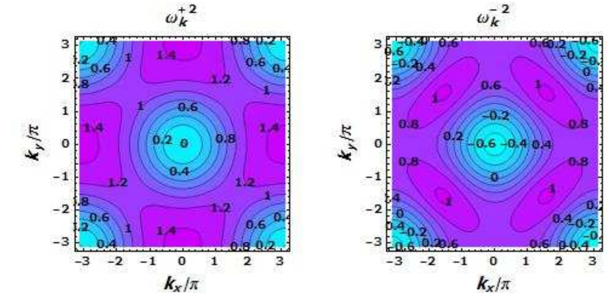

FIG. 2: (Color online) ω_k^{+2} and ω_k^{-2} in the π-flux phase when $\lambda = \Delta - \sqrt{2}\chi/2$ with $\Delta = 1.2021$ and $\chi = 0.2349$, where there exists a contour for the quasiparticle spectrum to cross from positiveness to negativeness in ω_k^{-2}.

For the π-flux phase, there are two branches of bosons, ω_k^+-boson and ω_k^--boson. Spinon condensation arises in each branch when

(1) $\lambda = \Delta + \sqrt{2}\chi/2$. At the wave vector $k^* = (0, 0)$ we obtain $\omega_{k^*}^+ > 0$ but $\omega_{k^*}^- = 0$. The self-consistent equation (17) deduces $\chi = 0$ corresponding to the lowest energy, which is the same as the uniform-flux case, and π-flux doesn't give any particular effect. Néel ordering occurs only for the ω_k^- branch.

(2) $\lambda = \Delta - \sqrt{2}\chi/2$. ω_k^+-branch bosons are condensed to the $k^* = (0, 0)$ state, which implies that collinear antiferromagnetic ordering is also realized within ω_k^+ bosons. On the other hand, if we examine the dispersion of low lying excitations of ω_k^- bosons, we find $\omega_k^- \approx \sqrt{2}\Delta/2\left(|k|^2 - 4\sqrt{2}\chi/\Delta\right)^{1/2}$ around the zero point. That is, there is a ring divergence rather than points provided that $\chi \neq 0$ (see Fig. 2). Thus, for the ω_k^- branch the bosons with $|k|^2 \leq 4\sqrt{2}\chi/\Delta$ are all going into the condensate portion of the boson state. In other words, the density of ω_k^- bosons stays at $|k|^2 \leq 4\sqrt{2}\chi/\Delta$ in momentum space, thus many kinds of non-collinear magnetic ordering emerge. Actually, we obtain a non-zero $\chi = 0.2349$ and $\Delta = 1.2021$, solving Eq.(17) numerically. Moreover, comparing with the ground state energy of the uniform-flux phase Eq.(18), this complex magnetic state is found to have a lower energy indeed,

$$E_\pi = -\Delta^2 + \chi^2 = -0.3578. \tag{19}$$

We can extract the collinear staggered magnetization from the ω_k^+ branch, and find $m_0 = 0.1043$. This value is much smaller than that of the uniform-flux case owing to enhancement of ferromagnetic correlations.

B. Discussion

In the previous section we have seen that ferromagnetic correlations do not contribute to physics of collinear antiferromagnetism in the conventional Schwinger-boson

mean-field approximation, i.e., uniform-flux ansatz. On the other hand, the π-flux phase turns out to be more stable than the uniform-flux phase, and the emergence of π-flux enhances ferromagnetic correlations, causing a complex magnetic state, where there exist various ordering wave vectors.

Nature of the magnetically ordered state in the π-flux phase is an important question. The present semiclassical approach does not seem to be sufficient for determining the resulting ground state. This situation reminds us of "order-by-disorder",[19] where quantum fluctuations split degeneracy and determine its ground state. One possible scenario in the square lattice is that magnetic ordering is still to show collinear antiferromagnetism despite the presence of such "degenerate" states, i.e., many k^*s corresponding to ordering vectors, since magnetism measured from the degenerate states will be cancelled out after momentum integration if the condensation amplitude at each momentum point is assumed to be the same as each other. These contributions reduce the strength of the collinear antiferromagnetism, as seen in the previous calculation. This is an important effect of ferromagnetic fluctuations in the antiferromagnetic ground state.

Existence of the complex spin texture prohibits emergence of gapless U(1) gauge fluctuations owing to nonzero vacuum expectation values of both spinon-hopping and spinon-pairing order parameters. Instead, Z_2 gauge theory appears for possible non-magnetic phases resulting from this complicated spin pattern.[5] Then, deconfined spinon excitations are expected to appear at high energies beyond their excitation gap. Actually, dynamic spin-excitation spectra have shown that there is large spectral weight at high energies beyond the spin-wave approximation in undoped cuprates.[20] In the present context this unidentified spectral weight may be identified with deconfined spinon continuum allowed by Z_2 gauge theory.

However, it should be noted that the Z_2 gauge structure is allowed only in the low energy limit, more precisely, $T < T_\chi$, where the ferromagnetic exchange-hopping parameter has its nonzero expectation value below T_χ. Then, U(1) gauge fluctuations appear above this temperature. The Z_2-U(1) crossover makes deconfinement of spinons not clear. This suggests to measure dynamic spin spectra carefully at high energies as a function of temperature although it is beyond the scope of the present mean-field study to predict this temperature accurately.

III. DOPED ANTIFERROMAGNETIC MOTT INSULATOR

A. A slave-fermion mean-field theory of the t-J model

1. A slave-fermion effective action

We start from the t-J model for doped Mott insulators

$$H = -t \sum_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + H.c.) + J \sum_{ij} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j). \tag{20}$$

Based on the U(1) slave-fermion representation

$$c_{i\sigma} = \psi_i^\dagger b_{i\sigma} \tag{21}$$

with the single occupancy constraint $\sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} + \psi_i^\dagger \psi_i = 2S$ where S is a value of spin, here $S = 1/2$, one can rewrite the spin-exchange term and electron hopping term in terms of fermionic holon and bosonic spinon operators as follows

$$\begin{aligned} & J \sum_{ij} (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j) \\ & \rightarrow \frac{J}{2} \sum_{ij} |\Delta_{ij}^b|^2 - J \sum_{ij} (\Delta_{ij}^{b\dagger} \epsilon_{\alpha\beta} b_{i\alpha} b_{j\beta} + H.c.), \\ & -t \sum_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + H.c.) \rightarrow t \sum_{ij} (\chi_{ji}^\psi \chi_{ij}^b + H.c.) \\ & -t \sum_{ij} (b_{i\sigma}^\dagger \chi_{ij}^b b_{j\sigma} + H.c.) + t \sum_{ij} (\psi_j^\dagger \chi_{ji}^\psi \psi_i + H.c.), \end{aligned} \tag{22}$$

where the bond operators

$$\Delta_{ij}^b = \sum_{\alpha\beta} \epsilon_{\alpha\beta} b_{i\alpha} b_{j\beta}, \quad \chi_{ji}^\psi = \sum_\sigma b_{i\sigma}^\dagger b_{j\sigma}, \quad \chi_{ij}^b = \psi_i \psi_j^\dagger \tag{23}$$

describe antiferromagnetic, ferromagnetic correlations, and hopping of doped holes, respectively. Comparing with the Heisenberg Hamiltonian [Eq. (9)], one can see $-t\chi_{ij}^b$ and $J\Delta_{ij}^b$ correspond to $J\chi_{ij}$ and $J\Delta_{ij}$, respectively.

Using the above expressions, the t-J model in the electron language is mapped onto exactly the same model but with fractionalized particles such as spinons and holons in the following way

$$\begin{aligned} L_{eff} &= \sum_i b_{i\sigma}^\dagger (\partial_\tau - \mu) b_{i\sigma} - t \sum_{ij} (b_{i\sigma}^\dagger \chi_{ij}^b b_{j\sigma} + H.c.) \\ & - J \sum_{ij} (\Delta_{ij}^{b\dagger} \epsilon_{\alpha\beta} b_{i\alpha} b_{j\beta} + H.c.) + \frac{J}{2} \sum_{ij} |\Delta_{ij}^b|^2 \\ & + \sum_i \psi_i^\dagger \partial_\tau \psi_i + t \sum_{ij} (\psi_j^\dagger \chi_{ji}^\psi \psi_i + H.c.) + t \sum_{ij} (\chi_{ji}^\psi \chi_{ij}^b + H.c.) \\ & + i \sum_i \lambda_i (b_{i\sigma}^\dagger b_{i\sigma} + \psi_i^\dagger \psi_i - 2S), \end{aligned} \tag{24}$$

where λ_i is a Lagrange multiplier field to impose the single occupancy constraint.

It is important to notice that ferromagnetic exchange correlations in the spin-exchange term is not allowed in the hole-doped case. This implies that at half-filling, this effective Lagrangian reduces to the conventional Schwinger-boson theory instead of the "flux" Schwinger-boson theory since the absence of holons results in $\chi_{ij}^b = 0$, causing ferromagnetic correlations to vanish, i.e., $\chi_{ij}^\psi = 0$. The reason why we do not introduce the ferromagnetic correlation term from the Heisenberg term is that when holes are doped, the presence of charge contribution, $-\frac{1}{4}n_i n_j$, does not allow the simple ground-state-energy analysis even for the case of classical spins, performed in the study of the Heisenberg model, thus it is not obvious that introduction of such contributions lowers the ground state energy. Here, we focus on the hopping term of the t-J model instead, giving rise to ferromagnetic spin correlations induced by doped holes. This doping-induced ferromagnetic-correlation term will play an important role in the structure of magnetic ordering. As seen from its sign, opposite to the half-filled case, it will contribute even in the uniform-flux case, and modify the collinear antiferromagnetism drastically.

In the following we will consider the π-flux phase although we comment on the uniform-flux case briefly. There are two reasons for considering such a flux phase. The π-flux phase is time-reversal symmetry-preserving, thus physically allowed. More importantly, the π-flux ansatz gives rise to four hole pockets, consistent with the previous studies[8], where the spinon-holon exchange hopping term (Shraiman-Siggia term) is taken into account in the context of the perturbation theory, revealing that holons reside in the four diagonal points. Although we do not know the connection between our flux-phase result and the previous perturbation theories, the π-flux ansatz reproduces such hole pockets.[21] On the other hand, the uniform-flux phase does not allow the hole pockets.

2. π-flux ansatz

Four order parameters are assumed to be as follows for the mean-field analysis

$$\Delta_{ij}^b \to \Delta_b, \qquad i\lambda_i \to \lambda,$$
$$\chi_{ij}^b \to \chi_b e^{-i\phi_{ij}^b}, \qquad \chi_{ij}^\psi \to \chi_\psi e^{-i\phi_{ij}^\psi}, \tag{25}$$

decoupling the slave-fermion effective Lagrangian, where the spin-singlet pairing order parameter and Lagrange multiplier field are assumed to be uniform and real while the hopping parameters of spinons and holons have their phase contributions. A nonzero value of Δ_b corresponds to short-range antiferromagnetic correlation as discussed before, and a nonzero χ_b (χ_ψ) implies hole (spinon) mobility or ferromagnetic correlation, which deforms the

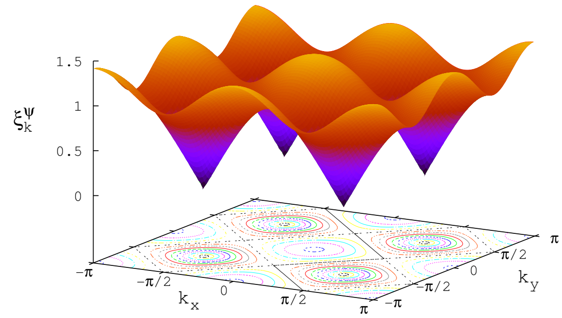

FIG. 3: (Color online) Energy dispersion of fermionic holes, where $Min[\xi_k^\psi] = 0$ results at $k = (\pm\pi/2, \pm\pi/2)$.

spin order in the presence of hole doping. Together with the π-flux ansatz

$$\chi_{ij}^b\chi_{jk}^b\chi_{kl}^b\chi_{li}^b = \chi_b^4 e^{-i\sum_\Box \phi_{ij}^b},$$
$$\chi_{ij}^\psi\chi_{jk}^\psi\chi_{kl}^\psi\chi_{li}^\psi = \chi_\psi^4 e^{-i\sum_\Box \phi_{ij}^\psi},$$
$$\sum_\Box \phi_{ij}^b = \sum_\Box \phi_{ij}^\psi = \pi \tag{26}$$

the free energy can be decomposed into the charge (fermion) and spin (boson) sectors,

(1) Fermion sector

$$F_\psi = -\frac{1}{\beta}\sum_{ks}{}' \ln\Big\{2\cosh\Big(\frac{\beta E_{ks}^\psi}{2}\Big)\Big\}$$
$$+2t\chi_\psi \sum_k{}' \sqrt{(\gamma_k\cos\Theta)^2 + (\varphi_k\sin\Theta)^2}$$
$$= -\frac{1}{\beta}\sum_k{}' \Big[\ln\Big\{2\cosh\Big(\frac{\beta}{2}[2t\chi_\psi\sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2}$$
$$+\lambda]\Big)\Big\} + \ln\Big\{2\cosh\Big(\frac{\beta}{2}[-2t\chi_\psi\sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2}$$
$$+\lambda]\Big)\Big\}\Big] + 2t\chi_\psi\sum_k{}' \sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2} \tag{27}$$

(2) Boson sector

$$F_B = \frac{1}{\beta}\sum_{ksl}^{\prime} \ln\Big\{2\sinh\Big(\frac{\beta E^b_{ksl}}{2}\Big)\Big\}$$
$$= \frac{2}{\beta}\sum_{k}^{\prime}\Big[\ln\Big\{2\sinh\Big(\frac{\beta}{2}\big\{\big(2t\chi_b\sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2}$$
$$-\mu+\lambda\big)^2-\big(4J\Delta_b\gamma_k\big)^2\big\}^{1/2}\Big)\Big\}$$
$$+\ln\Big\{2\sinh\Big(\frac{\beta}{2}\big\{\big(-2t\chi_b\sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2}$$
$$-\mu+\lambda\big)^2-\big(4J\Delta_b\gamma_k\big)^2\big\}^{1/2}\Big)\Big\}\Big], \quad (28)$$

where

$$\gamma_k=\cos k_x+\cos k_y,\quad \varphi_k=\cos k_x-\cos k_y,\quad 4\Theta=\pi. \quad (29)$$

Noted from the energy dispersion $\xi^{\psi}_k = \sqrt{(\gamma_k\cos\Theta)^2+(\varphi_k\sin\Theta)^2}$ of fermions (Fig. 3), one can see holes are pocketed around the diagonal points, $(\pm\pi/2,\pm\pi/2)$.

3. Self-consistent equations

In this study we focus on the properties at zero temperature. Minimizing the free energy with respect to Δ_b, χ_b, χ_ψ, λ and μ, we obtain the following self-consistent equations,

(1) Fermion sector

$$\delta = \frac{1}{2N_L}\sum_{\xi^{\psi}_k\le-\mu/2t\chi_\psi} 1 \quad (30)$$

$$\chi_b = -\frac{1}{4N_L}\sum_{\xi^{\psi}_k\le-\mu/2t\chi_\psi}\xi^{\psi}_k \quad (31)$$

Above equations clearly show that holons are most densely located within the Dirac cones at $k = (\pm\pi/2,\pm\pi/2)$. For small doping we have $\delta=\frac{1}{2\pi}\left(\frac{\mu}{2t\chi_\psi}\right)$ and $\chi_b=-\frac{\sqrt{2\pi}}{3}\delta^{3/2}$, as derived in appendix.

(2) Boson sector

$$\frac{1}{2N_L}\sum_{k,s=\pm}^{\prime}\frac{-s\xi^{\psi}_k\big(\lambda+2st\chi_b\xi^{\psi}_k\big)}{E^b_s(k)}=\chi_\psi \quad (32)$$

$$\frac{1}{N_L}\sum_{k,s=\pm}^{\prime}\frac{8J\Delta_b\gamma_k^2}{E^b_s(k)}=\Delta_b \quad (33)$$

$$\frac{1}{N_L}\sum_{k,s=\pm}^{\prime}\frac{\big(2st\chi_b\xi^{\psi}_k+\lambda\big)}{E^b_s(k)}=(2S+1)-\delta \quad (34)$$

with the spinon-quasiparticle spectrum $E^b_s(k)=\sqrt{\big(2st\chi_b\xi^{\psi}_k+\lambda\big)^2-\big(4J\Delta_b\gamma_k\big)^2}$. It is clear that these saddle-point equations are reduced to the conventional Schwinger-boson mean-field equations in the zero-doping limit ($\delta\to0$) since χ_b vanishes from Eq. (31) and χ_ψ disappears accordingly from the summation for $s=\pm$ in Eq. (32). Hole doping effects reduce the amplitude of an effective spin from $2S$ to $2S-\delta$, as shown in Eq. (34), and increase ferromagnetic correlations, as shown in Eq. (32). Accordingly, antiferromagnetic correlations become weaken, seen from Eq. (33).

In the mean-field treatment magnetic ordering is described by condensation of Schwinger bosons. Here, we introduce χ^{ψ}_{BC}, Δ^b_{BC} and n_{BC} to describe the contribution from the $E^b_s(k^*)=0$ states, which are taken into account separately:

$$\frac{1}{2N_L}\sum_{k,s=\pm}^{\prime}\frac{-s\xi^{\psi}_k\big(\lambda+2st\chi_b\xi^{\psi}_k\big)}{E^b_s(k)}$$
$$+\chi^{\psi}_{BC}\sum_{k^*,s=\pm}\frac{-s\xi^{\psi}_{k^*}\big(\lambda+2st\chi_b\xi^{\psi}_{k^*}\big)}{\lambda}=\chi_\psi \quad (35)$$

$$\frac{1}{N_L}\sum_{k,s=\pm}^{\prime}\frac{8J\Delta_b\gamma_k^2}{E^b_s(k)}+\Delta^b_{BC}\sum_{k^*,s=\pm}\frac{8J\Delta_b\gamma_{k^*}^2}{\lambda}=\Delta_b \quad (36)$$

$$\frac{1}{N_L}\sum_{k,s=\pm}^{\prime}\frac{\big(2st\chi_b\xi^{\psi}_k+\lambda\big)}{E^b_s(k)}+n_{BC}\sum_{k^*,s=\pm}\frac{\big(2st\chi_b\xi^{\psi}_{k^*}+\lambda\big)}{\lambda}$$
$$=(2S+1)-\delta. \quad (37)$$

If $E^b_s(k^*)=0$ and $\sum_{k^*,s=\pm}n_{BC}(2st\chi_b\xi^{\psi}_{k^*}+\lambda)/\lambda$ is finite, spinons are condensed to the $E^b_s(k^*)=0$ states, and the system possesses a magnetic long-range order. The ordering wave vector k^* is determined by $E^b_s(k^*)=0$, and especially, $k^*=(0,0)$ corresponds to long-range anti-ferromagnetic ordering. For definiteness, we have checked that our treatment of these mean-field equations recovers the uniform-flux Schwinger-boson mean-field theory correctly. At half-filling the spinon spectrum of $E^b_+(k)=E^b_-(k)$ is gapless at $k=(0,0)$ which ensures their Bose condensation. Solving Eq. (37), we see that the spontaneous staggered magnetization is given by $\langle S^z_i\rangle = n_{BC}\approx 0.3$, recovering the half-filled result completely.

4. Phase diagram

Now, let's consider how the antiferromagnetic long-range order evolves with doping. Due to the π-flux order, there are two types of bosons, B_+ with energy $-E^b_+(k)$ and B_- with energy $-E^b_-(k)$. Away from half-filling holes are constrained within the Dirac cones, thus Eq. (31) gives $\chi_b\le0$, which deduces B^b_+ has lower energy than B^b_- at finite doping, i.e., $-E^b_+(k)<-E^b_-(k)$. When $\lambda=8J\Delta_b+2\sqrt{2}t\chi_b$ is satisfied, we have $E^b_-(0)=0$ for B_- bosons, thus their Bose condensation results at the wave vector $k^*=(0,0)$, implying an antiferromagnetic

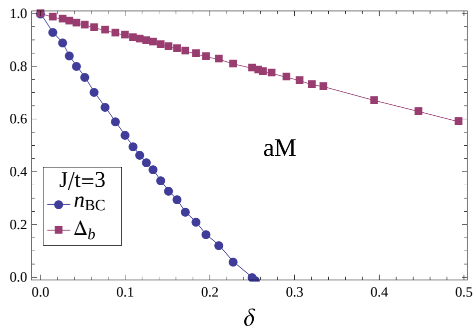

FIG. 4: (Color online) Doping dependence of Δ_b and n_{BC} with $t/J = 3$.[22] In order to clearly show the tendency, Δ_b and n_{BC} are scaled with the values at half-filling. The spin-gapped anomalous metallic phase is marked as aM.

order. For B_+ bosons within the same value of λ, one can easily check that $1/E^b_+(k^*)$ shows divergence along a ring $k^* \in \{k|E^b_+(k) = 0\}$. In other words, the density of B_+ spinons resides at large number of k^* states, thus many kinds of incommensurate magnetic ordering emerge and such a complex spin configuration deviates from the collinear antiferromagnetic ordering, similar to the half-filling case in the mean-field approximation.

With increasing doping and t, electrons become more active, implying that ferromagnetic correlations described by χ_b and χ_ψ should also increase. However, the antiferromagnetic order represented by the condensation amplitude n_{BC} becomes more suppressed with large t/J, which is confirmed by the numerical results. Furthermore, by numerically solving the self-consistent equations for given value of $t/J = 3$, Fig.4 exhibits that antiferromagnetic ordering disappears completely above $\delta_c \approx 0.25$[22] while the spin-singlet order persists up to much larger hole concentration ($\Delta_b \neq 0$) away from the antiferromagnetic order. We identify this intermediate non-magnetic spin-gapped phase with an anomalous metal, as will be discussed in the next section.

Compared to the uniform-flux case although not shown in the present paper, we have qualitatively the same phase diagram. This originates from the fact that the sign of the ferromagnetic correlation term is negative opposite to the Heisenberg model, thus such contributions are energetically favorable even in the uniform-flux phase. However, the presence of the π-flux order enhances ferromagnetic correlations as seen in the study of the Heisenberg model. As a result, the collinear antiferromagnetic order is killed more rapidly via hole doping in the π-flux phase. We have also examined the role of frustrated hopping effects, i.e., next-nearest-neighbor hopping terms. Such frustrated hopping effects modify bare dispersions for spinons and holons. However, we do not find any qualitative changes in the phase diagram since the presence of π-flux dominates over the band-modification effect.

B. Spin-gapped anomalous metal

As shown in the phase diagram of Fig. 4, hole doping kills the antiferromagnetic order and results in a paramagnetic state, where bosonic spinons are gapped but short-range antiferromagnetic correlations still remain. We identify this phase with an anomalous spin-gapped metal since charge excitations carried by holons are gapless, but spin fluctuations are gapped, thus exhibiting spin-charge separation.

To understand physics of this anomalous metallic phase, it is necessary to derive an effective field theory. Considering holons first, one can expand the holon field as

$$\begin{aligned}\Psi(x) &= e^{ik_1x}\psi_1(x) + e^{-ik_1x}\psi_{\bar{1}}(x) \\ &+ e^{ik_2x}\psi_2(x) + e^{-ik_2x}\psi_{\bar{2}}(x)\end{aligned} \quad (38)$$

around the diagonal points $k_1 = (\pi/2, \pi/2)$ and $k_2 = (\pi/2, -\pi/2)$, where the holon spectrum is linearized as

$$E^\psi_k = 2t\chi_\psi\sqrt{\cos^2 k_x + \cos^2 k_y} \approx v_\psi\sqrt{k_x^2 + k_y^2} \quad (39)$$

with its velocity $v_\psi = 2t\chi_\psi$. The holon spinor is given by

$$\psi_1(x) = \begin{pmatrix} \psi_{1e}(x) \\ \psi_{1o}(x) \end{pmatrix}, \qquad \psi_2(x) = \begin{pmatrix} \psi_{2o}(x) \\ \psi_{2e}(x) \end{pmatrix}, \quad (40)$$

where e and o represent even and odd sites, respectively. Such two-component spinors can be combined to form a single four-component spinor as

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = \begin{pmatrix} \psi_{1e}(x) \\ \psi_{1o}(x) \\ \psi_{2o}(x) \\ \psi_{2e}(x) \end{pmatrix}. \quad (41)$$

An important point is that U(1) gauge fluctuations become gapped due to condensation of a "charge" 2 scalar field, here the ferromagnetic hopping parameter, reduced to gapped Z_2 gauge fields, as discussed previously. In other words, hole doping causes magnetic frustration effects, giving rise to ferromagnetic correlations. This reduces the emergent U(1) gauge structure down to Z_2. As a result, we find the following effective field theory for the anomalous Z_2 spin-gapped metal phase

$$\mathcal{L}_\psi = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi - \mu_r\bar{\psi}\gamma_0\psi, \quad (42)$$

where Dirac gamma matrices satisfying the Clifford algebra $[\gamma_\mu, \gamma_\nu]_+ = 2\delta_{\mu\nu}$ are given by[23]

$$\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \; \gamma_1 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \; \gamma_2 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix},$$

and μ_r is a renormalized chemical potential associated with hole doping. A_μ is an electromagnetic field. Note that gapped spinon excitations are ignored for low energy

physics of this metallic phase. Then, Eq. (42) is our effective field theory for the anomalous metallic state.

An immediate question is about the transport property in this metallic phase. Since there is no scattering mechanism with gapless gauge fluctuations in the above effective field theory, Fermi liquid behavior is expected to appear in the low temperature limit, i.e., $\delta\rho(T) = \rho(T) - \rho(T=0) \sim T^2$ with resistivity $\rho(T)$. Although this is true in the present description actually, such a Fermi liquid behavior in transport is not a full story.

As discussed in the Heisenberg model, there is an energy scale T_χ associated with the crossover behavior from Z_2 to U(1) since ferromagnetic contributions are nonzero below this temperature. Although we cannot determine this temperature satisfactorily in our mean-field analysis, gapless U(1) gauge fluctuations should arise above this crossover temperature. Then, scattering with such gapless gauge excitations would lead to non-Fermi liquid physics in this regime. Frankly speaking, there are various regimes at finite temperatures, associated with not only the Z_2-U(1) crossover temperature but also so called the spin-gap temperature T_b, where bosonic spinons are effectively critical above this temperature. T_b should not be confused with T_Δ, where spinon-singlet pairing order appears below this temperature, much larger than the spin-gap temperature T_b. In this respect we should consider both energy scales, and classify various finite temperature regimes. In the present paper we discuss only low energy physics of this metallic phase.

IV. DISCUSSION AND SUMMARY

A. Coherent electron quasiparticle excitations near the diagonal points

The present slave-fermion mean-field description cannot explain the emergence of sharply defined electron quasiparticles near the diagonal points and their Fermi liquid behavior because electrons decay into spinons and holons in the Z_2 anomalous metal phase.[24] This is the main criticism in this kind of decomposition approach if one tries to connect the spin-charge separation scenario with pseudogap physics of high T_c cuprates. However, if we consider the regime of $T > T_\chi$, the situation is not perfectly clear owing to the presence of gapless gauge fluctuations beyond the mean-field description since such gauge fluctuations cause attractive interactions between holons and spinons, binding them, which may allow a resonance state corresponding to an electron-like excitation although it differs from confinement. Actually, this was a little bit demonstrated, where noncompact gauge fluctuations give rise to attractive interactions between "particle" and "hole" (holon and spinon in the electron spectrum), increasing coherence.[26] We should confess that this argument is far from consensus even in the qualitative point of view.

In the low temperature regime, more precisely in the zero temperature limit where we have studied in this paper, the situation becomes worse because even gapless U(1) gauge fluctuations do not exist owing to the gauge-symmetry reduction. However, we argue that such an electron excitation may still appear as a resonance state between a spinon and holon. To see this resonance state, we revisit the hopping-t term in the slave-fermion representation of the t-J model. We rewrite the hopping term as follows

$$
\begin{aligned}
&-t\sum_{ij}(b_{i\sigma}^\dagger f_i f_j^\dagger b_{j\sigma} + H.c.) \rightarrow t\sum_{ij}(c_{i\sigma}^\dagger c_{j\sigma} + H.c.) \\
&-t\sum_{ij}(c_{i\sigma}^\dagger f_j^\dagger b_{j\sigma} + H.c.) - t\sum_{ij}(b_{i\sigma}^\dagger f_i c_{j\sigma} + H.c.) \\
&+t\sum_{ij}(\chi_{ji}^\psi \chi_{ij}^b + H.c.) - t\sum_{ij}(b_{i\sigma}^\dagger \chi_{ij}^b b_{j\sigma} + H.c.) \\
&+t\sum_{ij}(\psi_j^\dagger \chi_{ji}^\psi \psi_i + H.c.). \qquad (43)
\end{aligned}
$$

It should be noted that the electron operator $c_{i\sigma}$ is different from the original one since it is a Hubbard-Stratonovich field although it is a fermion. Rather, it should be understood as a collective fermionic excitation in the spinon-holon liquid. We identify this fermion field as an electron, which is expected to be seen in the low energy regime. Note that hopping of such low energy electrons is frustrated, viewed from its sign.

The electron-spinon-holon exchange term reminds us of the slave-fermion representation of the Anderson lattice model, where localized spins are represented in slave-fermions.[27] To find a self-energy correction from this "hybridization" coupling seems to be an interesting and important problem since the spin-charge separation scenario may explain the coherent nodal electron excitations.

B. d-wave superconductivity

Another interesting subject is how to describe d-wave superconductivity in this context. Considering the slave-boson case, one may regard this study as a straightforward job. In the slave-boson theory condensation of bosonic holons results in superconductivity in the presence of spinon singlet pairing since they form Cooper pairs. However, this structure cannot be applied to the slave-fermion context. Fist of all, charged holons are fermionic, thus their condensation is forbidden. Then, the next choice will be pairing of such fermionic holons. Actually, such pairing interactions are usually attributed to the presence of gapless U(1) gauge fluctuations since they cause attractive interactions between holons living in different sublattices. What we would like to do is to construct a mean-field theory for such pairing order from the microscopic model itself. However, if we start from the t-J model, such an attractive pairing interaction channel is difficult to find since the exchange term

is represented only in the bosonic-spinon language owing to the single occupancy constraint, analogous to the fact that the J term is also expressed in terms of only fermionic spinons in the slave-boson context.

To avoid this difficulty, we start from so called BCS-Hubbard model[28]

$$H = -t\sum_{ij}(c_{i\sigma}^{\dagger}c_{j\sigma} + H.c.) - \sum_{ij}(\Delta_{ij}^{\dagger}\epsilon_{\alpha\beta}c_{i\alpha}c_{j\beta} + H.c.) + \frac{1}{g}\sum_{ij}|\Delta_{ij}|^2 + U\sum_{i} n_{i\uparrow}n_{i\downarrow} \quad (44)$$

where the Hubbard-Stratonovich transformation for the Cooper channel is performed before the constraint is applied. This model has its own interest in the fact that we can investigate what happens in BCS superconductivity, increasing local interactions U. Our strategy is that such strong correlation effects are introduced using the slave-fermion representation in the large-U limit. Then, the pairing term is expressed in the slave-fermion representation as

$$\begin{aligned} &-\sum_{ij}(\Delta_{ij}^{\dagger}\epsilon_{\alpha\beta}c_{i\alpha}c_{j\beta} + H.c.) \\ &= -\sum_{ij}(\Delta_{ij}^{\dagger}\epsilon_{\alpha\beta}b_{i\alpha}b_{j\beta}\psi_i^{\dagger}\psi_j^{\dagger} + H.c.) \\ &\to \sum_{ij}(\Delta_{ij}^{\dagger}\Delta_{ij}^{\psi}\Delta_{ij}^{b\dagger} + H.c.) - \sum_{ij}(\Delta_{ij}^{\dagger}\Delta_{ij}^{b\dagger}\epsilon_{\alpha\beta}b_{i\alpha}b_{j\beta} + H.c.) \\ &-\sum_{ij}(\Delta_{ij}^{\dagger}\Delta_{ij}^{\psi}\psi_i^{\dagger}\psi_j^{\dagger} + H.c.), \quad (45) \end{aligned}$$

where pairing interactions are renormalized due to Hubbard interactions. Replacing $\Delta_{ij}^{\dagger}\Delta_{ij}^{b\dagger}$ and $\Delta_{ij}^{\dagger}\Delta_{ij}^{\psi}$ with Δ_{ij}^{b} and Δ_{ij}^{ψ}, respectively, we find the slave-fermion effective Lagrangian for the BCS-Hubbard model in the large-U limit

$$\begin{aligned} L_{eff} &= \sum_{i} b_{i\sigma}^{\dagger}(\partial_\tau - \mu)b_{i\sigma} - t\sum_{ij}(b_{i\sigma}^{\dagger}\chi_{ij}^{b}b_{j\sigma} + H.c.) \\ &- \sum_{ij}(\Delta_{ij}^{b\dagger}\epsilon_{\alpha\beta}b_{i\alpha}b_{j\beta} + H.c.) \\ &+ \sum_{i}\psi_i^{\dagger}\partial_\tau\psi_i + t\sum_{ij}(\psi_j^{\dagger}\chi_{ji}^{\psi}\psi_i + H.c.) \\ &- \sum_{ij}(\Delta_{ij}^{\psi}\psi_i^{\dagger}\psi_j^{\dagger} + H.c.) \\ &+ \sum_{ij}(\frac{\Delta_{ij}^{\psi\dagger}\Delta_{ij}^{b\dagger}}{\Delta_{ij}} + H.c. + \frac{1}{g}|\Delta_{ij}|^2) + t\sum_{ij}(\chi_{ji}^{\psi}\chi_{ij}^{b} + H.c.) \\ &+ i\sum_{i}\lambda_i(b_{i\sigma}^{\dagger}b_{i\sigma} + \psi_i^{\dagger}\psi_i - 1). \quad (46) \end{aligned}$$

Although investigation of this effective Lagrangian is beyond the scope of this paper, one can read an important aspect of this Lagrangian immediately. An observed dome-shaped superconductivity line is expected to appear since Δ_{ij}^{b} vanishes in the zero doping limit owing the absence of doped holes, causing $\Delta_{ij}^{\psi} = 0$ while Δ_{ij}^{ψ} disappears at large doping, killing Δ_{ij}^{b}.

C. Summary

In this paper we have considered dynamics of doped holes in the antiferromagnetically correlated spin background. The point is that doped holes frustrate the collinear antiferromagnetic spin configuration, resulting in more complex spiral-like spin patterns. This reduces the U(1) gauge symmetry down to Z_2, thus the effective field theory appears to be a Z_2 gauge theory instead of U(1) in the low energy limit. As a result, deconfinement of spinons and holons is naturally allowed in $(2+1)D$. When spinons become gapped via further hole doping, an anomalous metallic phase results with spin-charge separation. We argued that there is a crossover energy scale T_χ from Z_2 to U(1) above this temperature, where transport should be more carefully studied in the presence of such gauge interactions. We also discussed how coherent electron excitations near the diagonal points can appear in the slave-fermion context. Lastly, we have speculated superconductivity within the slave-fermion representation.

APPENDIX: ANALYTIC EXPRESSIONS FOR FERMION SELF-CONSISTENT EQUATIONS NEAR ZERO DOPING

In this appendix we show the analytic expression of χ_b in the low doping limit. Minimizing the free energy Eq. (27) for χ_ψ, we find an equation for χ_b and obtain the

following expression

$$
\begin{aligned}
\chi_b &+ \frac{1}{2N_L}\sum_k{}' \xi_k^\psi = \frac{1}{8N_L}\Big[\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi \\
&- \sum_{\xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi - \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} \xi_k^\psi \\
&+ \sum_{\xi_k^\psi \geq (\mu+\lambda)/2t\chi_\psi} \xi_k^\psi\Big] = \frac{1}{8N_L}\Big[\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi \\
&+\Big(\sum_{(\mu+\lambda)/2t\chi_\psi \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi + \sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi\Big) \\
&- \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} \xi_k^\psi - \Big(\sum_{(\mu+\lambda)/2t\chi_\psi \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi \\
&+ \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} \xi_k^\psi\Big)\Big] \\
&= \frac{1}{4N_L}\Big[\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi - \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} \xi_k^\psi\Big] \\
&= \frac{1}{4N_L}\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi \\
&= \frac{1}{4N_L}\Big[\sum_{\xi_k^\psi \geq 0} \xi_k^\psi - \sum_{\xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \xi_k^\psi\Big] \\
&= \frac{1}{2N_L}\sum_k{}' \xi_k^\psi - \frac{1}{2} \times 4 \times \int_{\xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \frac{d^2k}{(2\pi)^2}\xi_k^\psi \\
&= \frac{1}{2N_L}\sum_k{}' \xi_k^\psi - \frac{1}{2\pi^2}\Big[\pi\Big(\frac{\mu+\lambda}{2t\chi_\psi}\Big)^3 - \frac{1}{3}\pi\Big(\frac{\mu+\lambda}{2t\chi_\psi}\Big)^3\Big] \\
&= \frac{1}{2N_L}\sum_k{}' \xi_k^\psi - \frac{1}{3\pi}\Big(\frac{\mu+\lambda}{2t\chi_\psi}\Big)^3 \qquad \text{(A.1)}
\end{aligned}
$$

with $\xi_k^\psi \equiv \sqrt{\cos^2 k_x + \cos^2 k_y}$.

In the same way we find the following expression from the variation of the free energy for the chemical potential μ

$$
\begin{aligned}
-\delta &= \frac{1}{4N_L}\Big[\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} - \sum_{\xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \\
&+ \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} - \sum_{\xi_k^\psi \geq (\mu+\lambda)/2t\chi_\psi}\Big] \\
&= \frac{1}{4N_L}\Big[\sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi} - \Big(\sum_{(\mu+\lambda)/2t\chi_\psi \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \\
&+ \sum_{\xi_k^\psi \geq -(\mu+\lambda)/2t\chi_\psi}\Big) + \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi} \\
&-\Big(\sum_{(\mu+\lambda)/2t\chi_\psi \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} + \sum_{\xi_k^\psi \leq (\mu+\lambda)/2t\chi_\psi}\Big)\Big] \\
&= -\frac{1}{2N_L}\sum_{(\mu+\lambda)/t\chi_\psi \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \\
&= -\frac{1}{2N_L}\sum_{0 \leq \xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \\
&\approx -\frac{1}{2} \times 4 \times \int_{\xi_k^\psi \leq -(\mu+\lambda)/2t\chi_\psi} \frac{d^2k}{(2\pi)^2} \\
&= -\frac{1}{2\pi^2} \times \pi\Big(\frac{\mu+\lambda}{2t\chi_\psi}\Big)^2. \qquad \text{(A.2)}
\end{aligned}
$$

As a result, we obtain

$$\chi_b = -\frac{2\sqrt{2\pi}}{3}\delta^{3/2}. \qquad \text{(A.3)}$$

In the uniform-flux case we find $\chi_b \propto -\delta^2$ smaller than the above in the zero doping limit

[1] In the heavy-fermion problem it is well known that the slave-boson approach describes the heavy-fermion Fermi-liquid phase quite well, but has difficulty in explaining antiferromagnetism while the slave-fermion approach captures the antiferromagnetic state well, but has difficulty in discussing the heavy Fermi-liquid phase. This is deeply related with statistics of spinons.

[2] Recently, one of the authors has derived an effective field theory of the slave-fermion spirit from the SU(2) slave-boson theory, where an SO(5) Wess-Zumino-Witten description for spin fluctuations and non-relativistic fermion U(1) gauge theory for dynamics of doped holes naturally emerge from fermionizing SU(2) slave bosons. See Ki-Seok Kim, arXiv:0804.0895 to be published in Phys. Rev. B.

[3] It will be an interesting problem to find a parameter discriminating symmetric Mott insulators from symmetry-broken ones, which may be analogous to the Ginzburg-Landau parameter distinguishing type I superconductors from type II ones.

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[21] One can understand the emergence of hole pockets from the Luttinger theorem, stating that the volume of a Fermi surface equals to the number of fermions inside the Fermi surface. Considering the generalized single-occupancy constraint with the presence of spinon-singlet pairs (Δ_i) in the slave-fermion representation,

$$\sum_{\sigma} b_{i\sigma}^{\dagger} b_{i\sigma} + 2\Delta_i^{\dagger}\Delta_i + f_i^{\dagger} f_i = 1,$$

we find that an electron number operator can be written as follows

$$N_{el} = \sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} = \sum_{\sigma} b_{i\sigma}^{\dagger} f_i f_i^{\dagger} b_{i\sigma} = f_i f_i^{\dagger}(1 - f_i^{\dagger} f_i - 2\Delta_i^{\dagger}\Delta_i)$$
$$= (1 - 2\Delta_i^{\dagger}\Delta_i)(1 - f_i^{\dagger} f_i).$$

As a result, we obtain the Luttinger theorem

$$\frac{V_{FS}^f}{(2\pi)^d} = 1 - \frac{N_{el}}{1 - 2|\Delta|^2},$$

where V_{FS}^f is the volume of the holon Fermi surface and $|\Delta|^2$ is the condensation amplitude of spinon pairs. When $|\Delta|^2 << 1$ is satisfied, one can expand the denominator, then we find in the zeroth-order approximation for Δ

$$\frac{V_{FS}^f}{(2\pi)^d} = 1 - N_{el} = \delta.$$

Thus, the volume of the holon Fermi surface should be proportional to hole concentration, explaining why hole pockets appear at low doping. However, the presence of spinon-singlet pairs reduces the volume of the holon Fermi surface, smaller than hole concentration δ.

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